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Helical and Spiral Antennas as Electric Small Antennas

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ABSTRACT
The simplest form of a classical antenna is the dipole antenna, which is essentially two pieces of wire placed end to end with a feed point in the middle. The length of this antenna is typically half the wavelength of the signal that is being received or transmitted. An "electric small" antenna is defined as an antenna of much shorter dimensions than the wavelength of the signals it is designed for. Small electric antennas have an advantage when space is the most essential factor. Satellites and mobile communication apparatuses for example can use small antennas in order to free up more space for other components. The problem with classical electric small antennas is that their bandwidth and radiation efficiency shrink as they get shorter. Although such antennas have been in use for decades, they remain difficult to design and limited in their applicability. However, the classic approach of studying metal antennas using only Maxwell equations limits the real internal nature of them as metal lattice structures and thus as quantum wells of free electrons. When interpreted this way, free electrons are obeying the Schrodinger wave equation and this view can give new ideas for designing effective curvilinear electric small antennas. Such a class of electric small helical and spiral antennas is proposed in the following paper.

Keywords: electric small antenna, antenna as quantum trap, curvilinear antennas, helical antennas, spiral antennas.

INTRODUCTION

It is well known that given the electric current of any antenna for any operating frequency, the electromagnetic field (near and far) can be calculated directly from Maxwell equations. Even if we remain in the class of the simplest kind of antennas, which are the one-dimensional center-fed linear wire antennas of an arbitrary length, only numerical methods exist for finding their approximate electric current distribution [1]. However, in the case that the length L of a thin center-fed rectilinear antenna is equal to $L=n*\lambda/2$, where λ is the operating wavelength of the electromagnetic wave with n an integer, its electric current can be accurately calculated and being given by a sinusoidal standing wave of wavelength λ and frequency C/λ (where C is the speed of light).

My personal view is that all the impressive progress in the field of linear antennas is mostly due to laborious experimental work rather than a clear theoretical understanding of their internal operation. This is the result of trying to describe this operation as the effect of action of electromagnetic fields on them, considering the antennas as simple non active conducting structures and thus ignoring their internal natural dynamics as quantum wells for free electrons. In the present paper we focus on thin linear antennas comprising a standard metallic lattice although an extension of the proposed method to more complicated structures is possible.

It will be shown in the present report that the optimum operation of any type of metallic lattice linear antenna can be achieved when the internal spatial waveforms of their free electron density become resonant with the externally applied electromagnetic fields, following their quantum mechanical behavior as described by the respective Schrodinger equation. An initial presentation of antennas as quantum wells of free electrons has already been published by the author [2].

Small electric antennas [3] are defined as the antennas that can operate effectively although their dimensions are much smaller than the wavelength of their operation. It is suggested that in order to design antennas for specific applications, like small electric antennas, we should take into consideration their internal nature as quantum wells of free electrons governed by the Schrodinger wave dynamics. A theoretical approach combining Schrodinger and Maxwell equations in order to understand the properties of curvilinear antennas is presented in the paper with the aim to design "small electric" antennas.

RECTILINEAR METALLIC ANTENNAS AS QUANTUM WELLS OF FREE ELECTRONS

Here, an example of a one-dimensional rectilinear antenna is studied as a quantum well of free electrons. The most common antenna of this kind is a center-fed linear metallic antenna of length L. Even the tinny metallic antennas of a few mm long have a tremendous number of free electrons. Thus, we can consider all the metal lattice rectilinear antennas as wells of a cloud of free electrons obeying the Quantum Mechanical laws as described by the respective Schrodinger equation.

Let us consider a rectilinear antenna taken as very thin in comparison to its length. Thus, it can be considered as a one-dimensional metal lattice structure along the x-axis from 0 to L. The cloud of free electrons is obeying the stationary Schrodinger wave equation given by:

$$\partial^2 Y(x)/\partial^2 x = -(\varepsilon - U) \cdot Y(x) \qquad (1a)$$

$$(\varepsilon - U) = \frac{2m}{(h/2\pi)^2} * (E - V(x)) \qquad (1b)$$

V(x) is the electrostatic potential acting on the free electrons and E is the energy of the system necessary to form the respective stationary Schrodinger wave. In fact, there is an internal electrostatic action on free electrons arising from the ions of the metallic lattice, however, this potential action can be omitted and the overall action due to the scattering of free electrons with the positive lattice ions is usually replaced by an equivalent Ohmic resistance.

In the classic Born interpretation of the Schrodinger equation, the function $|Y(x)^2|$ defines the probability of an electron being at point x. Due to the extremely high number of free electrons, we can assume that $|Y(x)^2|$ is proportional to the number of electrons at x, thus this function defines the electric charge along the conducting linear structure Q(x) and as a result, the function $Y(x)\partial Y(x)/\partial x$ is proportional to the electric current of the linear antenna EC(x).

The eigenvalues and eigenfunctions of linear antennas will then be given by the solutions of the Schrodinger equation $\partial^2 Y(x)/\partial^2 x = -\varepsilon \cdot Y(x)$, i.e. the functions without any external voltage excitation. These eigenfunctions should be a set of eigenstates $Yn(x)$ with respective eigenvalues ε_n defined by the geometric boundaries of the curvilinear structure, where the respective electric current function I(x), is taken to be zero. Thus, at x=0 and x=L, either Y(x)=0 or $\partial Y(x)/\partial x = 0$

Let us assume that the boundary conditions for a certain curvilinear antenna are Y(0)=0 and Y(L)=0, hence the respective eigenstates are:
$Yn(x) = An\, sin(2m\pi x/2L)$

while for boundary conditions Y(0)=0 and $\partial Y(L)/\partial x = 0$, the respective eigenstates should be:
$Yn(x) = An\, sin((2m+1)\pi x/2L)$

Thus, for both cases $Yn(x) = An\, sin(n\pi x/2L)$ where n is an even or an odd integer the electric current I(x) in the linear antenna will be given as $I(x) = In * sin\,(n\pi x/L)$. For both cases the respective eigenvalues i.e. the energy of the system for each eigenstate will also be given as ε_n= *(nπ/2L)^2.* In a similar way, it can be proven that for $\partial Y(0)/\partial x = 0$ and *Y(L)=0* or $\partial Y(L)/\partial x = 0$ the electric current and the respective energy eigenvalues remain the same as previously.

The fundamental eigenstate for the electric current is the one where the minimum energy eigenvalue is achieved. Thus, the fundamental eigenstate is achieved for n=1, where the energy is minimum and equal to $\varepsilon_1 =(\pi/2L)$^2, and the respective electric current in the rectilinear antenna is $I(x)=I_1* sin\,(\pi x/L)$.

Thus, the lowest eigenstate for a linear center-fed antenna of length L is achieved by a semi sinusoidal current wave that becomes zero at the endpoints and maximum at its center, where we place the feed points of the antenna. This antenna will be tuned by an internal voltage source (transmitter) or by an external Electromagnetic field (receiver), with a sinusoidal voltage of angular frequency ω=k*C where k is equal to the wave number of its fundamental eigenstate π/L and C=speed of light, at which state it will generate its highest output.

In general, a linear bipolar antenna of length L can be also tuned in higher frequencies ω_n=n*ω where tuning is achieved for higher harmonics, k_n =(n*π)/L and ω_n =k_n*C. Any proper center fed bipolar antenna is tuned with odd harmonics that give a maximum output at their centers, while the even harmonics give zero output in their centers. Bipolar antennas are not used for higher frequencies because they demand more energy (n^2 times) to transmit and stronger fields to receive.

Thus, the smallest tuned center-fed linear antenna has a length of half of the wavelength of the electromagnetic wave, where it is designed to operate i.e. L=λ/2. In the case of a conducting ground plane below the half of the antenna, its length could be L=λ/4. In a higher wavelength range (smaller frequencies than f), the rectilinear bipolar antenna has a deteriorating operation demanding far more energy in order to transmit or receive the respective electromagnetic signals. An electric small antenna is defined as one with substantially smaller dimensions in relation to its operating wavelength band operating with relatively small energy.

CURVILINEAR ANTENNA AS QUANTUM WELL OF FREE ELECTRONS

Let us now consider a curvilinear one-dimensional, very thin metallic antenna inside three-dimensional free space. The geometrical shape of the antenna can be defined as a function of its linear length s(x,y,z) as a parametric curve, where $L \geq s \geq 0$.

The geometry of the curvilinear metallic structure of the antenna defines the Schrodinger equation of the cloud of free electrons inside its metallic lattice. More accurately the Schrödinger wave equation for a one-dimensional (very thin) curvilinear conducting structure developed along its parametric length *s* as has been proved [4] to be of the form

$$\partial^2 Y(s)/\partial^2 s = -\left(\frac{\sigma^2(s)}{4} + \varepsilon\right) \cdot Y(s) \qquad (2)$$

Where *σ(s)* is the standard local curvature of the curvilinear one-dimensional antenna and $\varepsilon = \frac{2m}{(h/2\pi)^2} E$ is the reduced energy of the cloud of free electrons inside the metallic lattice of the antenna.

In equation (2) we have a boundary value problem for a second order ordinary linear differential equation with variable coefficient because its curvature factor is a function of s, however, there are several numerical methods that can tackle the problem.

In the present paper an effective technique named "resonance technique", used in many similar applications by the author [5,6] will be applied. The great advantage of the "resonance technique" method is that it calculates initially the eigenvalues of the second rank differential equation and afterwards the respective eigenstates i.e. the harmonics of the relative antenna are readily obtained.

It should also be mentioned that the harmonics of the antenna are derived independently of the properties (frequency f) of the acting electromagnetic field. However, for successful operation of the antenna the frequency of the electromagnetic field should be adjusted accordingly.
The method is shortly presented as follows.

The original ODE is mathematically equivalent and is transformed to the following system of two first order differential equations:

$$\partial V(s) / \partial s = -j * A(s) * I(s)$$
$$\partial I(s) / \partial s = -j * V(s)$$

Where the used functions are defined as: $V(s) = j * \partial Y(s) / \partial s$, $I(s) = Y(s)$ and $A(s) = \left(\frac{\sigma^2(s)}{4} + \varepsilon\right)$
These equations are equivalent to a set of two "spatial electric-lines" along s. The non-linear homogeneous system of differential equations has solutions (eigenstates) only for special values of the energy ε (energy eigenvalues), and the fundamental eigenstate is the one for the minimum energy eigenvalue.

The antenna length s can be separated in very small intervals by N+1 successive points $s_1,s_2,s_3,\ldots$. where the first point is at $s_1=0$ and the final point at $s_{N+1}=L$. The respective successive intervals are named, $ds_1=s_2-s_1$, $ds_2=s_3-s_2$, etc, while the final interval is the ds_N.

Taking into consideration the mathematical properties of the "spatial electric lines", in the case of a harmonic (eigenstate) the curvilinear structure will be in a kind of spatial resonance. Thus, in every point s_M of it the impedances calculated from the left and right should be imaginary and in case of the proper ε opposite numbers of zero sum.

We can calculate numerically the overall impedance in a middle point s_M of the curve as shown from the left and from the right, starting from the impedances in the terminal points s_1 and s_{N+1}, where the impedances are zero (in case that V(s) is zero) or an extremely high number (in case that I(s)=0), and applying the resonance condition for sufficiently small ds intervals in comparison to $\frac{\sigma^2(s)}{4}$, the recursive formula will be,

$$Z(s_{n+1}) = (Z(s_n) - j * A(s_n) * ds_n)/(-j * Z(s_n) * ds_n + 1) \qquad (3)$$

from s_1 to s_M and from s_{N+1} to s_M is obtained.

The derived impedances $Z(s_M) = V(s_M)/I(s_M)$ at s_M from the left and right recursions are imaginary numbers and their sum should be zero in case of resonance and the appearance of an eigenstate. If we scan numerically a reasonable interval D around zero, we can find the respective minimum positive energy eigenvalue ε_1.

Using the defined fundamental eigenvalue in the function $A(s_n)$, the eigenstate $Y(s_n)= I(s_n)$ and its derivative $\partial Y(s_n) / \partial s= V(s_n)/j$ at every point s can be calculated using the matrix formula

$$[V(s_{n+1}) \quad I(s_{n+1})] \approx \begin{pmatrix} 1 & -jA(s_n)ds_n \\ -jds_n & 1 \end{pmatrix} [V(s_n)\ I(s_n)\ ] \qquad (4)$$

The starting point is taken to be the terminal s_1 with the initial vector being taken as $[V(s_1)\ I(s_1)\ ] = [1 \quad 0\ ]$. That vector is related to the boundary conditions imposed on the procedure of calculating the eigenvalue.

Furthermore, the eigenstate electric current $EC(s_n)$ in every point of the curvilinear antenna can be calculated by the formula

$$I(s_n) = ECn\ |Y(x)\partial Y(x)/\partial x\ |= ECn\ |\ I(s_n)\ V(s_n)\ | \qquad (5)$$

The procedure with the respective MATLAB routines is presented in appendix B of the present paper.

CENTER-FED HELICAL METAL ANTENNAS

Helical antennas are already in use in several applications while a classic presentation of their properties and uses can be found in [7,8]. Our previous quantum mechanical analysis of a very thin curvilinear antenna can be used also for helical antennas as quantum wells of free electrons. Their study will be facilitated in case they have a constant curvature σ. In this case the factor

$A(s)= (\frac{\sigma^2(s)}{4} + \varepsilon)$ is a constant independent of s, along the curvilinear antenna extending from 0 to L.

The respective ODE system becomes a second-rank partial differential equation with constant coefficients

$$\partial^2 Y(s)/\partial^2 s = -A \cdot Y(s) \quad (5a)$$

$$A= (\frac{\sigma^2}{4} + \varepsilon) \quad (5b)$$

The functions $Y(s)$ should be a set of eigenstates $Yn(s)$ with respective eigenvalues ε_n defined by the geometric boundaries of the curvilinear antenna, where the electric current wave function is considered zero. Thus, at s=0 and s=L, either Y(s)=0 or $\partial Y(s)/\partial s = 0$. According to geometry, the only set of one-dimensional curves of non-zero constant curvature in a three-dimensional space is the set of helical curves.

The curved linear conducting helical antennas of constant curvature σ are in general given as functions of a parameter t (angle length), by the set of equations in Cartesian coordinates as

$$x(t) = a\,cos\,(t)\,, y(t) = a\,sin\,(t)\,, z(t) = b\,t, 0 \leq t \leq T$$

$$s(t) = t(a^2 + b^2)^{1/2} \;\; => \;\; L = T(a^2 + b^2)^{1/2} \quad (6)$$

The constant curvature σ of this helical wire is given by $\sigma = |a|/(a^2 + b^2)$

As a result, of the boundary conditions, following the analysis of the rectilinear antenna that has a constant curvature equal to zero, for any set of boundary conditions the harmonics of the electric current (EC) in the helical antenna will be given by

$$I(s) = In * sin\left(\frac{n\pi}{L} s\right), \;\; \left(\frac{\sigma^2}{4} + \varepsilon\right) = \left(\frac{a^2}{4(a^2+b^2)^2} + \varepsilon\right) = \left(\frac{n\pi}{2L}\right)^2 \quad (7)$$

Thus, the minimum energy for excitation of the fundamental harmonic it will be calculated by the relation

$$\varepsilon = \left(\frac{n\pi}{2L}\right)^2 - \frac{a^2}{4(a^2+b^2)^2} > 0 \quad (8)$$

Let us assume that the arc of the helix is given by an integral number of semi-circles T=k*π, where k is an integer, hence $L = \pi k (a^2 + b^2)^{1/2}$

$$\varepsilon = \frac{\left(\frac{n}{2k}\right)^2}{a^2+b^2} - \frac{a^2}{4(a^2+b^2)^2} > 0 \quad (10)$$

For a helical arc antenna of b << α, the minimum positive energy ε is achieved for n=k thus L=nπ and is then given as

$$\varepsilon = \left[\frac{b^2}{4(a^2+b^2)^2}\right] \sim 0 \quad (11)$$

Under these conditions the minimum energy of the fundamental eigenfunction can become negligible thus, the curved helical wire can be tuned by a small voltage potential and the respective fundamental harmonic electric current of the helical antenna of k semi-circles will be given by:

$EC(s) = I\ sin(t) \quad ,0 \leq t \leq n\pi$

Hence the fundamental harmonic arising from the Schrodinger quantum behavior of the free electron cloud inside the metallic lattice of the antenna is sinusoidal along s.

For a properly designed symmetric center-fed helical antenna, L should be of an odd number of semi-circles, k=2*m+1 and the feed points of the antenna will be at the center of the (m+1) semi-circle.

Up to this point, no mention has been made about the wavenumber and the respective frequency of the transmitted or received electromagnetic signal by the helical antenna. It is possible that at least an optimal case could be for the wavelength of the signal to be λ= π/(2L), but this optimality can be proved only experimentally.

CENTER-FED HELICAL ANTENNAS AS ELECTRIC SMALL ANTENNAS

As an example of the previous analysis let us consider a helical antenna with a radius a=1, b=0.02, thus slope a/b=50 and pitch 2πb. The overall length of this antenna is chosen to be an odd integer number of semi-circles i.e. $T = 11\pi$. Taking into consideration that the b is much smaller than the radius α the length of the antenna $L = T(a^2 + b^2)^{1/2} \approx T * a = 11\pi a$. This thin metallic helical antenna is shown in figure (a) which should be center-fed in our case.

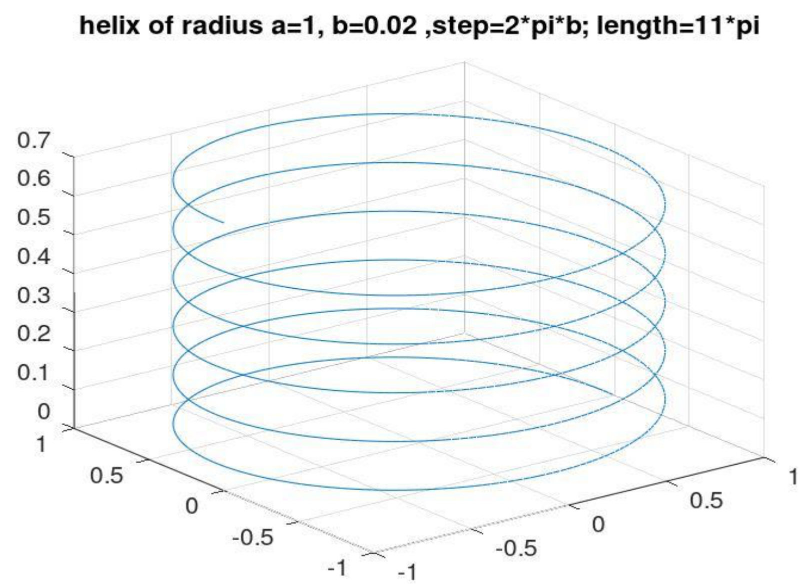

figure (a). A helical antenna of a=1,b=0.02, and arc-length T=11π.

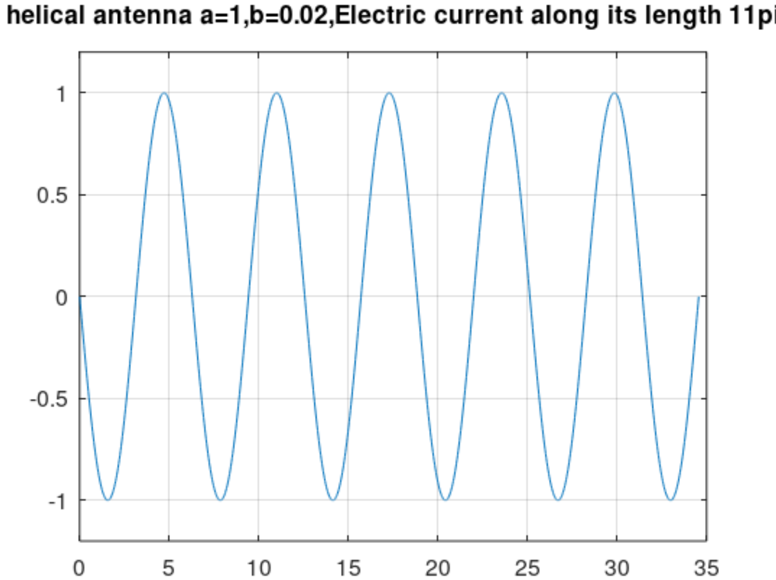

figure (β). Electric current along the length s of the helical antenna of fig(a) is shown, for its fundamental operation.

If this was a rectilinear center-fed antenna, the fundamental wavelength, i.e. the one with the minimum energy requirement to activate, would be $\lambda = L/2\pi$, and its operating frequency would be $f = C/2L$ where C is the speed of light. In this center-fed helical antenna, the fundamental wavelength along its length should be $\lambda = 2L$

However, no restrictions are necessary to be imposed on the helical antenna related to the operating frequency of the electromagnetic appropriate for its optimal operation. Given the fact that the minimum energy of the fundamental is almost zero, it is possible that the helical antenna to operate sufficiently with frequency $f = C/2L$. In this case, the helical antenna will be an electric small antenna where its dimensions approximately are defined by 2α, while the equivalent rectilinear antenna dimensions defined by its length will be ~17 times higher:

$$L = 5.5\pi(2a) \approx 17(2\alpha)$$

The proof of this argument should be experimentally tested, and due to its validity if true, will be necessary a proper laboratory be funded.

In any case, theoretical evidence points that this helical antenna could be a much better choice for this relatively low-frequency regime and the overall length L of the helix, in relation to the small linear bipolar antenna of the same dimensions (length ~2a).

SPIRAL PLANAR ANTENNAS

A similar quantum mechanical treatment of a spiral planar rectilinear antenna can be used in order to calculate its optimal dimensions for an operation as a small electric antenna. As is known there are many types of planar spiral antennas [9] and almost all of them have a variable curvature along length. In the present paper we are going to limit our study to the planar Archimedean spiral antennas considering that all the other planar spiral antennas can be studied in a similar way.

The Archimedean planar spiral antenna is given by the equation $r = a + b\theta$ where r,θ are its polar coordinates a is its initial radius and b is its pitch , $0 \leq \theta \leq T$. In the following figure an Archimedean planar spiral antenna of a=1, b=0.02 and overall arc-length 11π is given.

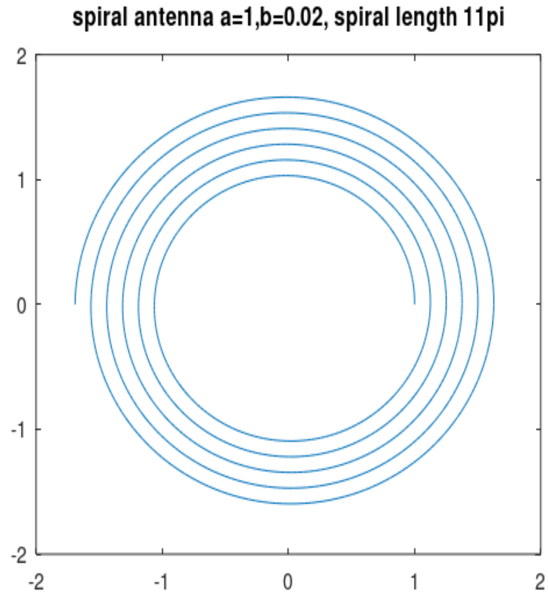

figure (c). An Archimedean spiral antenna of initial radius a=1, b=0.02, arc-length 11π.

The curvature of this planar spiral antenna in polar coordinates is given by $\sigma = (r^2 + 2b^2)/(r^2 + b^2)^{3/2}$ that for b<<a (very small in comparison to a) gives σ≈1/r. while its length s(θ) of this spiral is given by:

$$s(\theta) = \int_0^\theta \left(\sqrt{r^2 + b^2}\, d\theta\right), where\ for\ b^2 \ll r^2,\ s(\theta) \approx \int_0^\theta (r\, d\theta) = a\theta + 0.5b\theta^2$$

And $ds = \sqrt{r^2 + b^2}\, d\theta \approx r\, d\theta$

The length of the spiral in the figure .... for a=1, b=0.02, angular length T=11π units, will have an overall length L≈11π+0.12$(11\pi)^2$≈46.5 units.

Taking into consideration that the curvature of the planar spiral antenna is a function of r (thus a function of s) we should follow the numerical method of paragraph 3 in order to calculate numerically the fundamental (minimum) eigenvalue and its relative eigenfunction.

For the numerical calculation of the thin planar Archimedean spiral antenna will be divided in N equal arcs of $d\theta = 11\pi/N$, thus the respective intervals along s will be $ds(n) = \sqrt{r(n)^2 + b^2}\, d\theta$ And the respective local curvatures will be $\sigma(s_n) = (r(n)^2 + 2b^2)/(r(n)^2 + b^2)^{3/2}$

Thus, the set of respective differential equations, as shown in paragraph 3 of the present paper, will be:

$$\partial V(s) / \partial s = -jA(s)\, I(s)$$
$$\partial I(s) / \partial s = -j\, V(s)$$

Where, V(s)= $j\,\partial Y(s)\,/\,\partial s$, I(s)= $Y(s)$ *and* $A(s_n)$= $(\frac{\sigma^2((s_n)}{4}+\varepsilon)$

And $[V(s_{n+1})\quad I(s_{n+1})] \approx \begin{pmatrix} 1 & -jA(s_n)ds_n \\ -jds_n & 1 \end{pmatrix} [V(s_n)\quad I(s_n)\,]$

The fundamental minimum energy ε can be calculated by zeroing the function of the sum of impedances (see appendix B) and starting from the terminal s_1 =0, where the initial vector is $[V(s_1)\quad I(s_1)\,] = [1\quad 0\,]$, the vector functions $V(s_n)$ *and* $I(s_n)$ in the respective points are defined and finally the electric current (EC) in the spiral antenna is calculated as EC(s_n)= $ECn|V(s_n)I(s_n)|$

In figure (b) is shown the electric current of the internal fundamental harmonic along an Archimedes spiral antenna of a=1 and b=0.02 as calculated numerically using the MATLAB routines of appendix B.

As noticed also for the helical antennas, the frequency of the operating electromagnetic waves for spiral antennas is not defined by its internal characteristics. It is the author's belief that it is possible for an EM wave of wavelength near 2L to tune the spiral antenna, but this has to be proved experimentally. The feed points of the spiral antenna should be at the closest point to the middle, where the current becomes maximum. As shown in the figure (d) this point is at a distance of approximately 20 units starting from the initial inner edge of the spiral antenna that will have a total length of approximately 46.5 units.

The actual dimensions of this spiral planar antenna are approximately 16 times smaller than the length of an equivalent dipole antenna.

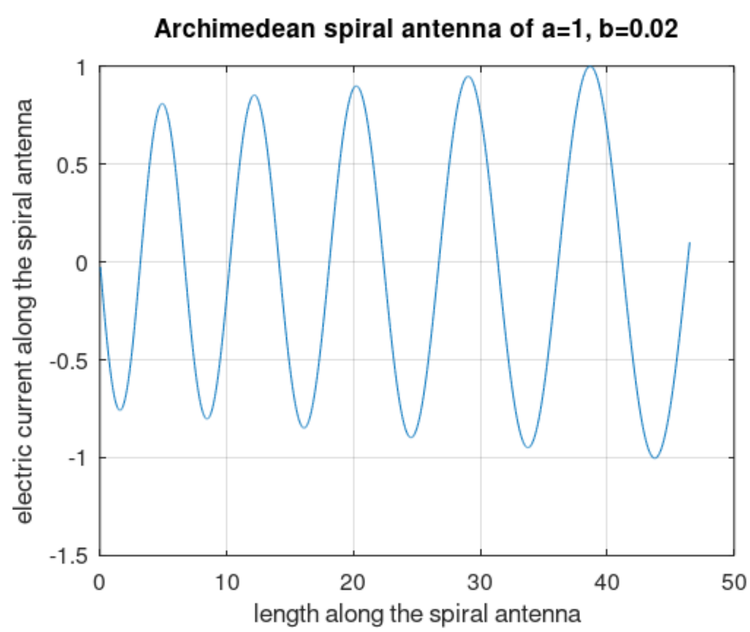

figure (d). The electric current along the length s of the spiral antenna shown in fig(c), for its fundamental operation.

SPATIAL FOURIER TRANSFORM ANALYSIS OF CURVILINEAR ANTENNAS

The hard problem related to curvilinear three-dimensional antennas is the calculation of their currents. According to theory the electromagnetic field generated by a thin three-dimensional curvilinear antenna in free space defined as a function of its linear length s(x,y,z) along it, where $L \geq s \geq 0$, of known electric current I(s) of frequency f can always be calculated. Here we use the method of calculating the far field radiation of a linear antenna using Fourier transform analysis [10] where the Spatial Fourier of the field components along x and y are utilized.

Any curvilinear thin antenna can be divided into N+1 successive points $s_1, s_2, s_3, \dots s_{N+1}$, thus can be considered approximately equivalent to a successive set of N very small linear dipoles $ds_1 = s_2 - s_1$, $ds_2 = s_3 - s_2$ ….., of known respective currents $I(s_n)$ where all the dipoles are very small and where the first point of the antenna is the s_1 and the final point s_{N+1}.

Let us consider the antenna as a transmitter. Every generated electromagnetic field component represented by a function $FF(x,y,z)exp\,(j\omega t)$ can be spatially Fourier transformed along x and y with wave numbers α and β thus $FF(x,y,z)\ exp\ exp\ (j\omega t)\ \to F(\alpha,\beta,z)exp\ (j\omega t)$.

Double initial letters represent the initial function while same single letters represent its spatial Fourier transform.

It can be proved that the partial differential Maxwell equations in free space, for the radiating far field waves are equivalent to two independent sets of ordinary differential equations as follows:

$$\frac{\partial Vm}{\partial z} = -\frac{c^2}{j\omega\mu} I_m \qquad \frac{\partial I_m}{\partial z} = -j\omega\mu V_m$$
$$\frac{\partial V_e}{\partial z} = -\frac{c^2}{j\omega\varepsilon} I_e \qquad \frac{\partial I_\varepsilon}{\partial z} = -j\omega\varepsilon V_e$$

Where: $c^2 = k^2 - (\alpha^2 + \beta^2)$, $k^2 = \varepsilon\mu\omega^2$

α, β, c and k are shown in figure…. , where k is the overall wave number of the electromagnetic field, $\omega = \frac{k}{C}$ with $C=1/\sqrt{\varepsilon\mu})$ = speed of light in empty space, $\varepsilon\omega = k/Zo$, $\mu\omega = k\ Zo$, Zo=120π and α, β, and c are the wavenumbers along the axis x,y and z.

by the initial Maxwell equation analysis can be shown that the functions *Vm, Im, Ve* and *Ie* (in Fourier space) are related with E_x, E_y, E_z , $H_x, H_y, H_{z,}$ which are the Fourier Transforms of the electric and the magnetic field components, by the following relations:

$Vm = aHx + \beta Hy$ $\qquad Ve = \alpha Ex + \beta Ey$

$Im = \beta Ex - \alpha Ey = \omega\mu\mathrm{Hz}$ $\quad Ie = \alpha Hx - \beta Hx = \omega\varepsilon E\mathrm{z}$

It can be proved furthermore that the radiating power (far field) in the free space above the horizon of any curvilinear antenna of known *Im , Ie,* can be calculated by the integral

$$P = \frac{1}{8\pi^2} \iint_{\lambda<\kappa} \frac{c}{\alpha^2 + \beta^2} \cdot \left(\frac{|I_e^2|}{\varepsilon\omega} + \frac{|I_m^2|}{\mu\omega}\right) d\alpha d\beta$$

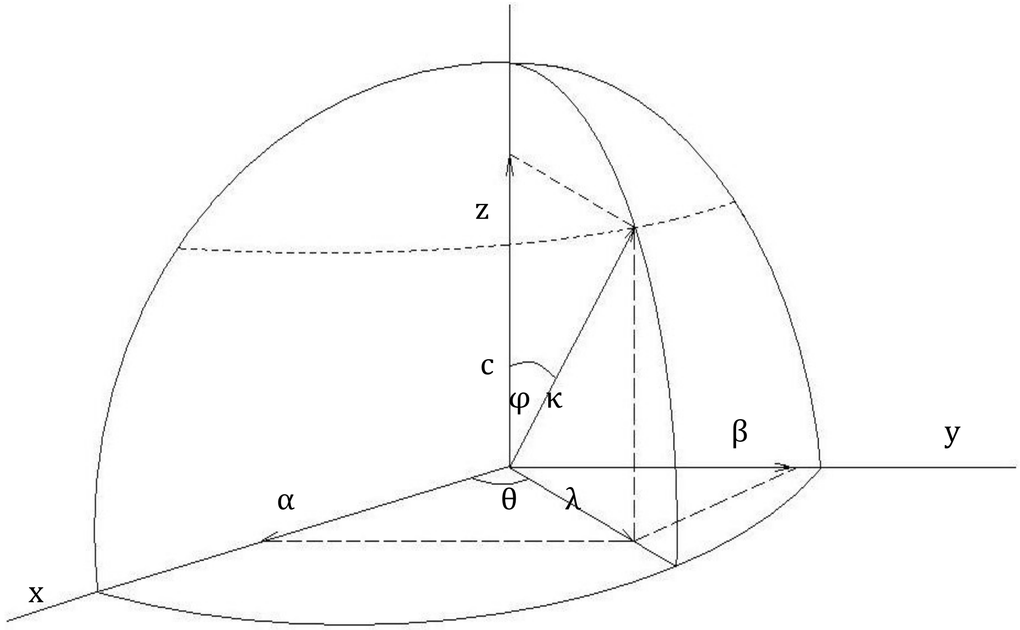

figure (e). Representative figure of the far field radiating components of an antenna above the horizon. k is the overall vector radiation, α,β,c its components along the axis x,y,z

Finally replacing, $c = k\cos\theta, \alpha^2 + \beta^2 = \lambda^2 = k^2\sin\theta^2, d\alpha d\beta = k^2\cos\theta\sin\theta\, d\theta\, d\varphi$, where k is the wavenumber of radiating field, the following integral is derived:

$$P = \frac{k}{8\pi^2}\int_0^{2\pi}\int_0^{\pi/2}\left(\frac{|I_\varepsilon(\theta,\varphi)|^2}{\varepsilon_0\omega} + \frac{|I_m(\theta,\varphi)|^2}{\mu_0\omega}\right)\left(\frac{\cos\theta}{\sin\theta}\right)^2 \sin\theta\, d\theta d\varphi$$

Also, taking into consideration that the $\sin\theta\, d\theta d\varphi$ is the surface spherical differential element on a sphere of radius k, the radiation pattern of the antenna is given by the following function:

$$F(\varphi,\theta) = \frac{1}{8\pi^2}\left(\frac{|I_\varepsilon(\theta,\varphi)|^2}{\varepsilon_0\omega} + \frac{|I_m(\theta,\varphi)|^2}{\mu_0\omega}\right)\left(\frac{\cos\theta}{\sin\theta}\right)^2$$

The functions $V_m(\alpha,\beta)$ and $V_\varepsilon(\alpha,\beta)$ can be calculated using the integrals along the curvilinear antenna as follows:

$$V_m(\alpha,\beta) = \int_s I(x,y,z)e^{j(\alpha x+\beta y+cz)}(\beta dx - \alpha dy)$$

$$Ve(\alpha,\beta) = \int_s \frac{I(x,y,z)}{\varepsilon_0\cdot\omega}\cdot e^{j(\alpha x+\beta y+cz)}[\alpha c dx + \beta c dy + (\alpha^2+\beta^2)dz]$$

These line integrals on the thin curvilinear antenna can be calculated numerically by dividing the antenna in N very small pieces with N+1 successive points $s_\kappa = (x_\kappa, y_\kappa, z_\kappa)$ where:

$\Delta x_\kappa = x_{\kappa+1} - x_\kappa$, $\Delta y_\kappa = y_{\kappa+1} - y_\kappa$, $\Delta z_\kappa = z_{\kappa+1} - z_\kappa$

$$G_\kappa = \frac{\alpha(x_\kappa + x_{\kappa+1})}{2} + \frac{\beta(y_\kappa + y_{\kappa+1})}{2} + \frac{c(z_\kappa + z_{\kappa+1})}{2}$$

For $\Delta x_\kappa, \Delta y_\kappa$, Δz_κ very small $G_\kappa \approx \alpha x_\kappa + \beta y_\kappa + c z_\kappa$

$V_m(\alpha,\beta) = \sum_{\kappa=1}^{N} exp\, exp\,(jG_\kappa)\ [\beta(\Delta x_\kappa) - \alpha(\Delta y_\kappa)]\, I_\kappa$

$V_e(\alpha,\beta) = \frac{1}{\varepsilon_0 \cdot \omega} \cdot \sum_{\kappa=1}^{N} \ exp\ exp\ (jG_\kappa)\ [\ \alpha c \Delta x_\kappa + \beta c \Delta y_\kappa + (\alpha^2+\beta^2)\Delta z_\kappa] I_\kappa$

In free space the respective functions $I_m(\alpha,\beta), I_\varepsilon(\alpha,\beta)$ can be calculated by the relations:
$I_e(\alpha,\beta) = V_\varepsilon(\alpha,\beta)/(2Z_\varepsilon)$, $\quad I_m(\alpha,\beta) = V_m(\alpha,\beta)/(2Z_m)$
Where $Z_e = c/\varepsilon\omega$, $\quad Z_m = c/\mu\omega$

Replacing α,β,c by their equivalent functions of θ,φ the function $\frac{|I_e(\theta,\varphi)|^2}{\varepsilon_0\omega} + \frac{|I_m(\theta,\varphi)|^2}{\mu_0\omega}$ can be calculated and the radiating far field power and its radiation pattern can be calculated numerically.

RADIATION PATTERNS OF A HELICAL AND SPIRAL ANTENNAS

Using the MATLAB codes shown in Appendices A and C the radiation patterns of the helical and spiral curvilinear antennas shown in fig (a) and (c) has been derived and are shown in the figures (ef),(g),(h), and (i)

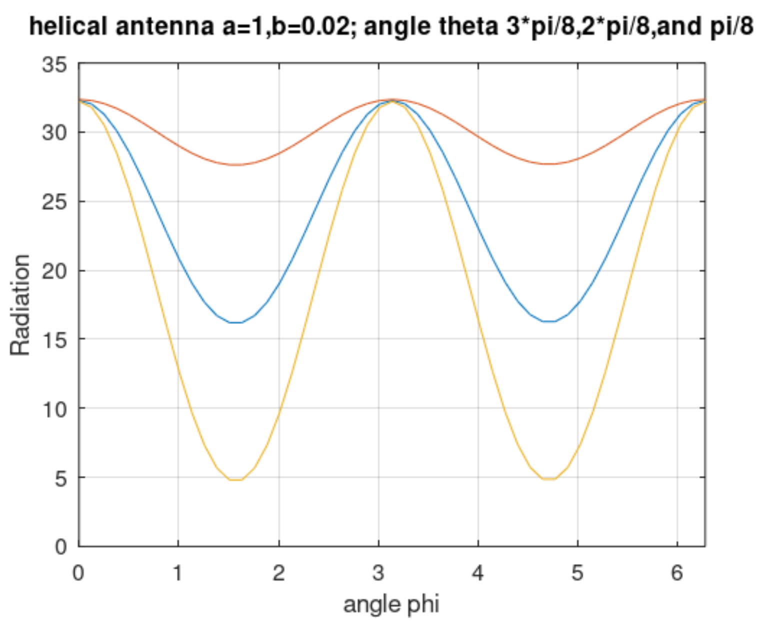

figure (f). Radiation pattern curves of the helical antenna of fig(a) for three values of θ and variable angle φ for its fundamental mode operation

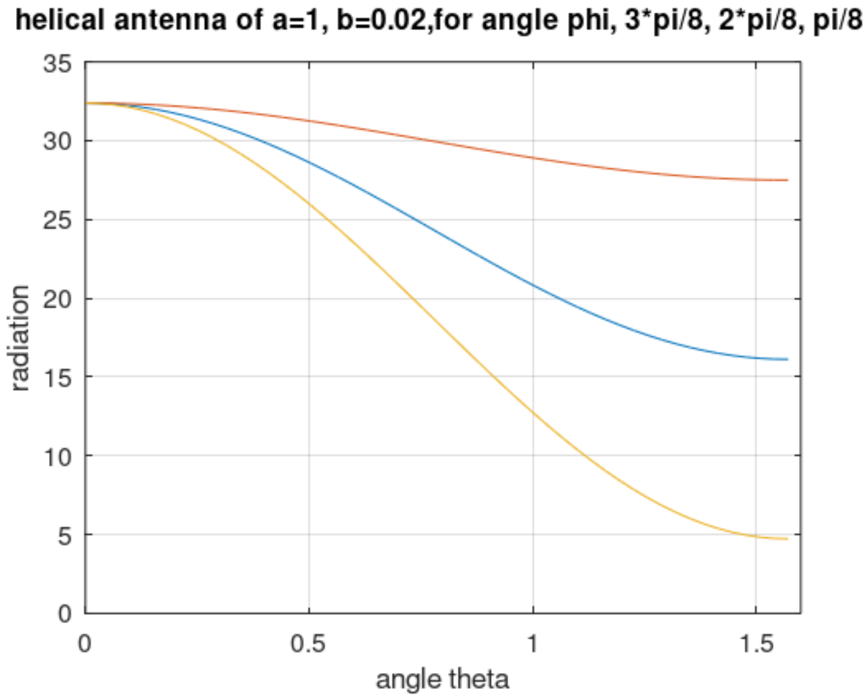

figure (g). Radiation pattern curves of the helical antenna of fig(a) for three values of φ and variable angle θ for its fundamental mode operation

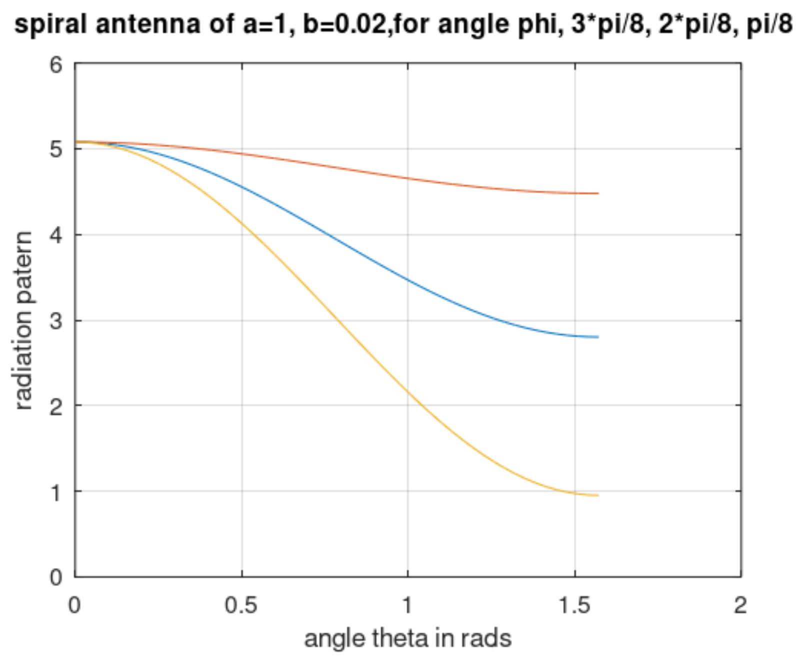

figure (h). Radiation pattern curves of the spiral antenna of fig(c) for three values of θ and variable angle φ for its fundamental mode operation

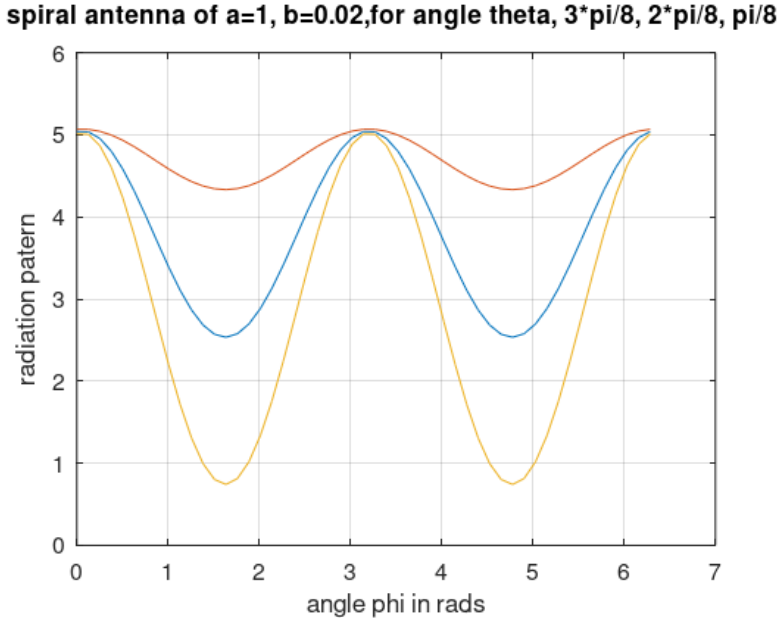

figure (i). Radiation pattern curves of the spiral antenna of fig(c) for three values of φ and variable angle θ for its fundamental mode operation

The given MATLAB codes can be used in any kind of curvilinear antennas in order to calculate their fundamental eigenvalues and their respective currents when are properly tuned by an external electromagnetic field of frequency f, and furthermore to calculate their radiation patterns. The purpose of presenting the MATLAB codes is to show that the mathematical analysis presented in the paper leads in relatively simple codes.

CONCLUSION

Although the antenna theory books and papers are full of cumbersome mathematical analyses, their whole methodology is based on the assumption that the antennas are non-active metal structures without internal dynamics. The only assumption made is that the electric field component is zero inside them (or normal to their surface).

However, in the author's view, tuned antenna properties are defined by the internal dynamics of the cloud of their free electrons obeying the Schrodinger equation. Thus, the study of tuned antennas should start from their inner dynamics and their optimal operation will appear whenever the externally imposed electromagnetic excitation has a proper frequency in resonance with the internal electron dynamics. Studying electric antennas solely from Maxwell's equations seems to be the same as studying springs or strings solely from the Applied Forces on them, ignoring their internal dynamics.

Hence, in the present work we assume that the metallic lattice of any antenna acts as a potential well of free electrons confined by its external surface and the cloud of them forms standing waves determined by the second-rank partial differential Schrodinger equations plus the respective boundary conditions (zero normal currents on their surfaces) with a frequency defined by the external electromagnetic field.

In the paper, the quantum well method is used in order to study thin curvilinear metallic antennas aiming into defining curvilinear configurations behaving as small electric antennas i.e. antennas where the diameter of the minimum geometric sphere containing the antenna is much smaller than the wavelength of their operation.

With the proposed method the operation of two curvilinear antennas one helical and one spiral was studied and we speculated that it is possible to be used as small electric antennas. The offered conclusion is that these antennas could possibly operate sufficiently with frequencies and wavelengths related to their total linear length which is much higher than the diameter of their containing geometric spheres.

Given further experimental verification of the validity of the above assumptions, it would enable the construction of similar small electric antennas of high operating wavelength. This means that many problems demanding small electric antennas can be solved with the use of special types of helical or spiral antennas or similar types of curvilinear antennas. Let us mention that Spiral antenna has, for several applications, the advantage of being flat and therefore more convenient to be used as a kind of patch antenna, stuck on a thin dielectric layer. It is obvious that these hypothetical conclusions urge for experimental confirmation, due to the importance of the subject [11].

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References

Hisamatsu Nakano, Low-Profile Natural and Metamaterial Antennas Analysis Methods and Applications, Hosei University, Koganei, Tokyo, 2016 John Wiley & Sons, Inc

Christos D. Papageorgiou, Curved Linear Antennas as Quantum Traps, International Journal of Communications http://www.iaras.org/iaras/journals/ijoc (2018)

Kyohei Fujimoto,Hisashi Morishita, Small Antennas, https://www.cambridge.org/core/books/modern- small antennas/B669EC0F559A67C7543B83BF9B56855B (2016)

J. Stockhofe and P. Schmelcher, Nonadiabatic couplings and gauge-theoretical structure of curved quantum waveguides, Physical Review A89 (033630) (2014).

C.D. Papageorgiou A.C. Boucouvalas, Propagation constants of cylindrical dielectric waveguides with arbitrary refractive index profile using Resonance technique, Electronic letter 2 September 1982 Vol 18 No 18

C. D. Papageorgiou A. D. Raptis, A method for solution of Schrodinger equation, Computer Physics Communication 43(1987)

Helical antenna https://en.wikipedia.org/wiki/Helical_antenna

Helical antenna theory https://www.antenna-theory.com/antennas/travelling/helix.php

Spiral antenna https://en.wikipedia.org/wiki/Spiral_antenna

C.D.Papageorgiou and J.D. Kanellopoulos, Equivalent circuits in Fourier space for the study of electromagnetic fields, j.Phys. A:Math. Gen. 15 (1982) 2569-2580, Great Britain

https://spectrum.ieee.org/electrically-small-antenna

APPENDIX A

MATLAB routine for calculation of radiation patterns of helical antennas

```
function P=wwhelix(ff,hh)
% radiation pattern calculation of a center fed helical antenna of arc 11*pi,
% wavelength kk=
% radius aa , slope bb, pitch 2*pi*bb, (antenna length)=L*sqrt(aa^2+bb^2)
% t is the variable defining the helix s=t*sqrt(aa^2+bb^2); divided in 1000 parts
% wave length=(antenna length)/pi, antenna electric current=Ic(s)=sin(s)
% ff=angle theta, hh=angle phi
global aa bb L
aa=1.; bb=0.02; L=11*pi
ab=sqrt(aa^2+bb^2);
Im=0; Ie=0;
for n=1:1001
dt=L/1000; t(n)=dt*(n-1);tt=t(n); x(n)=aa*cos(tt);y(n)=aa*sin(tt);z(n)=bb*tt;z0=120*pi;
ss(n)=tt*ab;s=ss(n);ddx(n)=-
aa*sin(tt)*dt;dx=ddx(n);ddy(n)=aa*cos(tt)*dt;dy=ddy(n);dz=bb*dt;
Ic(n)=sin(s);kk=pi/(L*ab);c=kk*cos(ff);a=kk*sin(ff)*sin(hh);b=kk*sin(ff)*cos(hh);
q(n)=a*x(n)+b*y(n)+c*z(n);
vm(n)=exp(j*q(n))*(a*dy-b*dx)*Ic(n);im(n)=vm(n)*z0/2/cos(ff);
ve(n)=exp(j*q(n))*Ic(n)/kk*z0*(a*c*dx+b*c*dy+(a^2+b^2)*dz);ie(n)=ve(n)/2/z0/cos(ff);
Im=Im+im(n);Ie=Ie+ie(n);
end
P=(1/8/pi^2)*(abs(Ie^2)/kk*z0+abs(Im^2)/kk/z0)*(cos(ff)/sin(ff))^2;
```

APPENDIX B

MATLAB routines for the calculation of currents of spiral antennas

```
function y=spir1(x)
% calculates the impedance in a middle point N/2
global aa bb
N=1000;aa=1;bb=0.02;
ss=0;
for n=1:N;
t=(n-1)*11*pi/N;dt=11*pi/N;
r(n)=aa+t*bb;
ku=(r(n)^2+2*bb^2)/(r(n)^2+bb^2)^1.5;
A(n)=(ku^2/4+x);
ss=ss+sqrt(r(n)^2+bb^2)*dt;
s(n)=ss;
ds(n)=sqrt(r(n)^2+bb^2)*dt;
end
z1=-j*10^10;z2=0;
for n=1:N/2;
z1=(z1-j*A(n)*ds(n))/(-j*z1*ds(n)+1);
z2=(z2-j*A(N-n)*ds(N-n))/(-j*z2*ds(N-n)+1);
end;
```

```
y=imag(z1+z2);
      % if y is zero x is eigenvalue
function y=eigspir
%Calculates the eigen function and the electric current of the spiral antenna as f(n)
ee=fzero(@spir1,0)
% ee is the minimum eigenvalue calculated by fzero near 0
 global N aa bb L
ss=0; aa=1.;b=0.02;N=1001;L=11*pi;
for n=1:N;
t=(n-1)*L/(N-1);dt=L/(N-1);
 r(n)=aa+t*bb;
 ku=(r(n)^2+2*bb^2)/(r(n)^2+bb^2)^1.5;
 A(n)=(ku^2/4+ee);
 ss=ss+sqrt(r(n)^2+bb^2)*dt;
 s(n)=ss;
 ds(n)=sqrt(r(n)^2+bb^2)*dt;
end
iv=[0;1];
for n=1:N-1
AA=[1          -j*A(n)*ds(n)          ;          -j*ds(n)          1];
iv=AA*iv;
f(n)=imag(iv(1))*real(iv(2));
s1(n)=s(n);
end
f=f/max(f);
plot(s1,f)
% The electric current (f(n)) of the spiral antenna along its length (s1(n))
```

APPENDIX C

MATLAB routine for the calculation of the radiation patterns of spiral antennas

```
function P=wwspiral(ff,hh)
global N aa bb L f
ss=0; aa=1.;bb=0.02;N=1001;L=11*pi;
 % radiation pattern calculation of a properly fed spiral antenna of arc 11*pi,
 % wavelength kk , initial radius of the spiral aa ,  pitch bb
 % t is the variable angle defining the spiral ; divided in N parts
 % wave length=(antenna length)/pi,
 % antenna electric current=EC(n)=f(n)
% ff=angle theta, hh=angle phi
 Im=0;Ie=0; z0=120*pi;kk=pi/L;
 for n=1:N-1
 dt=L/N; tt(n)=dt*n; t=tt(n); r=aa+bb*t; x(n)=r*cos(t);y(n)=r*sin(t);
 dx=(bb*cos(t)-r*sin(t))*dt ; dy=(bb*sin(t)+r*cos(t))*dt ;
Ic(n)=f(n); c=kk*cos(ff);
  a=kk*sin(ff)*sin(hh);b=kk*sin(ff)*cos(hh); q(n)=a*x(n)+b*y(n);
vm(n)=exp(j*q(n))*(a*dy-b*dx)*Ic(n);
  im(n)=vm(n)*z0/2/cos(ff);
 ve(n)=exp(j*q(n))*Ic(n)/kk*z0*(a*c*dx+b*c*dy);
 ie(n)=ve(n)/2/z0/cos(ff);
 Im=Im+im(n); Ie=Ie+ie(n);
end
 P=(1/8/pi^2)*(abs(Ie^2)/kk*z0+abs(Im^2)/kk/z0)*(cos(ff)/sin(ff))^2
```